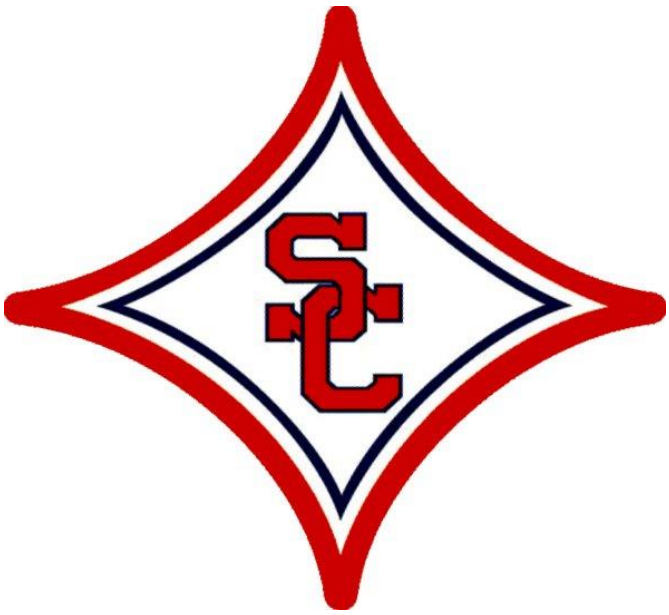
SANDY CREEK HIGH SCHOOL
SUMMER REVIEW PACKET

For students entering ***A.P. CALCULUS AB***

I expect everyone to check the Google classroom site and your school emails at least once every two weeks. You should also familiarize youself with the Khan Academy website before the school year starts.
By signing below, I acknowledge that I worked every problem on this assignment myself. While I was allowed to seek help from others, I did not copy someone else's work and submit it as if it was my own.

Printed Name: _____

Signature: _____ Date: _____



1. This packet is to be handed in to your Calculus teacher on the first day of the school year. You will also submist electronic copies of each page throughout the summer. The due dates for each page are listed on the page and must be submitted by midnight of that day to our Google classroom site.
2. All work must be shown **in the packet**, next to the problem. Circle final answers.
3. You are expected to do this packet **without a calculator**.
4. Completion of this packet is worth one-half of a major test grade. It will be graded as follows:
   Packet completed with all work shown = 25 points
   Five randomly selected problems graded for accuracy @ 5 points each = 25 points
5. You will have a multiple choice Calculus Readiness Test on the first day of the school year that also will be worth one-half of a major test grade.
6. You will have a quiz on the second day of the school year over your understanding of the unit circle that will be worth one quiz grade.

***Note: In calculus, you ALWAYS use RADIAN mode. No answers should ever be in degrees.**
***Note: Decimal answers should be accurate to three decimal places. However, since this is a no calculator packet, you should provide exact answers. For example: Write $\sqrt{3}$, not 1.732.**

Before each submission I will try to host a learning lunch at Partner's in Tyrone for you to ask questions. Please check the Google classroom site for times and date of those. Also, I am available through email kluemper.tony@mail.fcboe.org if you have any quesitons while you work.

Please submit to Google classroom by midnight on Friday, June 16

1. *Identify the following statements as true or false. If false, correct the statement or explain the error.*

Statement	True or False?	Correction/Explanation if false
$(x + 2)(x - 3) = 5$ $x + 2 = 5$ or $x - 3 = 5$ $x = 3$ or $x = 8$		
$\frac{3x-5}{6} = \frac{x-5}{2}$		
$\frac{14x-12w}{10+6y} = \frac{7x-6w}{5+3y}$		
$(x + 5)^2 = x^2 + 25$		
If $1 = \sqrt{2x+7} - \sqrt{x+3}$, then $1^2 = \left(\sqrt{2x+7}\right)^2 - \left(\sqrt{x+3}\right)^2$		
$\sqrt{x^2+49} = x+7$		
$\sqrt{49x^2} = 7x$		
$\sqrt{9} = \pm 3$		
If $x^2 = 9$, then $x = \pm 3$.		
$x^2 + x^2 = x^4$		
$\frac{x+5x^2}{x} = 5x$		
Solve for x: $x^2 = 4x$ I can divide both sides by x and I get $x = 4$.		
$f(x) = -x^2$ $f(-3) = -9$		

$f(x) = x^2$ $f(-3) = -9$		
$\left(3 \cdot w^3 \cdot x^4\right)^2 = 3w^6x^8$		
$\frac{y}{7x^{-1}} = 7xy$		
$2(3-g)(4+z) = (6-2g)(8+2z)$		
$2(5x) = (2 \cdot 5)(2 \cdot x)$		
ln(x) means the natural logarithm multiplied by x.		
sin(x) means the sine function multiplied by x.		
$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$		
$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$		
$\frac{(x-3)x - x^2(x+1)}{(x-3)} = x - x^2(x+1)$		
$\ln(x + e) = \ln(x) + \ln(e)$		
$2^{x+4} = 2^x + 2^4$		
$\cos^2 x = (\cos x)^2$		
$\cos^{-1} x = \sec x$		

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{3}$		
If I subtract $x - b$ from a, I get $a - x - b$.		
(5a)(5b) = 5(ab)		
$3(x + 6)^2 = (3x + 18)^2$		
$\frac{1}{3x}=\frac{1}{3}x$		
$\frac{x}{3}=\frac{1}{3}x$		

2. What is the point/slope form for the equation of a line? Use this form for questions 3-5.

3. Write the equation of the line whose slope is -4 and passes through (-3, 2).

4. Write the equation of the line parallel to $3x - 2y = 8$ that passes through the point (2, 5).

5. Write the equation of the line perpendicular to $3x - 2y = 8$ that passes through the point (2, 5).

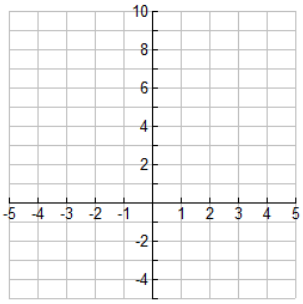
6. Find the average rate of change of $f(x) = x^2 + 6x - 10$ from $x = 2$ to $x = 4$.

7. Find $\frac{f(x+h)-f(x)}{h}$ for the given function f:

   a) $f(x)=9x+3$

   b) $f(x)=5-2x$

8. Graph $f(x)=\begin{cases} -1 & \text{if } x \le -2 \\ 1+x & \text{if } -2 < x \le 2 \\ 2x & \text{if } x > 2 \end{cases}$



Find the following:

f(-3) = _____ f(-2) = _____ f(-1) = _____ f(0) = _____ f(1) = _____ f(2) = _____ f(3) = _____

9. Let f(x) = $1 + \frac{1}{x}$ and g(x) = $\frac{3}{\sqrt{x}}$. Find f(g(x)).

10. Solve the system of equations: $\begin{aligned} 2x-3y &= -21 \\ 5x+6y &= 15 \end{aligned}$

11. Is the function even, odd, or neither. Justify your answer using f(x), f(-x) and –f(x).

a) $f(x)=\frac{x^4}{x^2-1}$

f(-x) = _____

-f(x) = _____

Even, odd, or neither? _____

b) $f(x) = x^3 + 2x^2 - 1$

f(-x) = _____

-f(x) = _____

Even, odd, or neither? _____

REMINDER NOTES FOR RATIONAL FUNCTIONS

Use the original problem for step 1.

1. Horizontal Asymptote / Slant Asymptote

- deg N < deg D — H.A. @ y = 0
- deg N = deg D — H.A. & y = ratio of leading coefficients
- deg N > deg D — no H. A.; Use long division to find slant asymp.

Factor the numerator and denominator, but do not cancel yet.

2. Domain

- Look at the denominator before you cancel and see what makes the bottom equal to zero.
- $\Re \neq$ those x-values

Now, cancel and simplify.

3. Holes

- If nothing cancels, there is not a hole.
- Set the canceled factor = 0. That is the x-coordinate of the hole.
- To get the y-coordinate, plug that x-value into the simplified function.

4. Vertical Asymptotes

- Set the denominator of the simplified function = 0 and find x.
- x = that value is the vertical asymptote

5. x-intercepts (zeros)

- Set the numerator of the simplified function = 0 and find x.
- (_____,0)

6. y-intercept

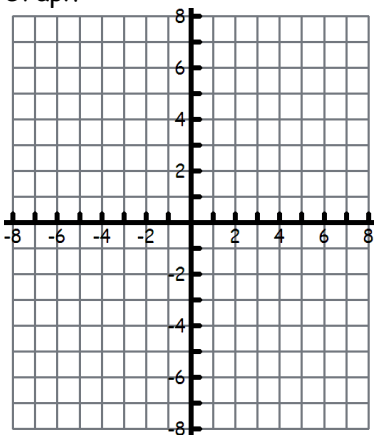
- Plug in zero for all the x's in the simplified function and evaluate it.
- (0,_____)

12. Identify all the necessary parts of the following rational functions and sketch it. Write "none" where applicable.

(a) $f(x) = \dfrac{3x^2 - 6x - 9}{x^3 - 9x}$

Horizontal Asymptote	
Slant Asymptote	
Hole(s)	
Vertical Asymptote(s)	
Domain	
x-intercepts	
y-intercepts	

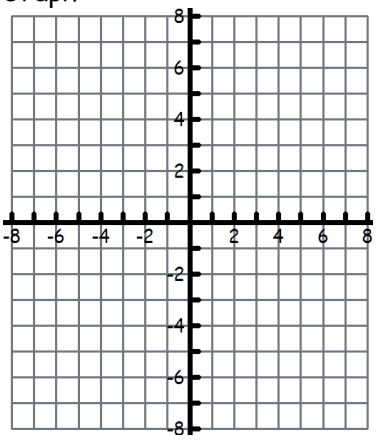
Graph



(b) $f(x) = \dfrac{2x^2 + 10x + 12}{x^2 + 3x + 2}$

Horizontal Asymptote	
Slant Asymptote	
Hole(s)	
Vertical Asymptote(s)	
Domain	
x-intercepts	
y-intercepts	

Graph



13. Simplify each expression.

a) $\dfrac{x^4 \cdot x^7}{x^6}$ b) $(s^4t^7)^3$ c) $\dfrac{x^2y^5}{(x^2)^4}$

14. Evaluate each expression.

a) $\left(27a^6\right)^{\frac{1}{3}}$ b) $27^{\frac{2}{3}}$ c) $\left(\dfrac{3}{4}\right)^{-1}$

15. Write each equation in exponential form.

a) $\log_{27} 3 = \dfrac{1}{3}$ b) $\log_{16} \dfrac{1}{4} = -\dfrac{1}{2}$

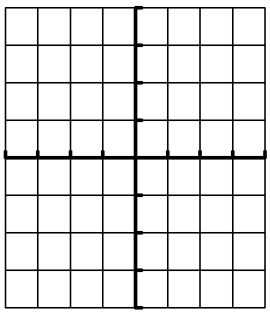
16. Write each equation in logarithmic form.

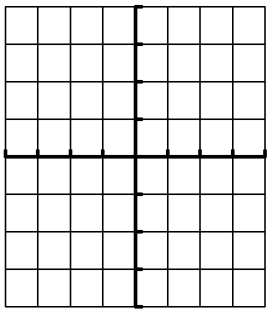
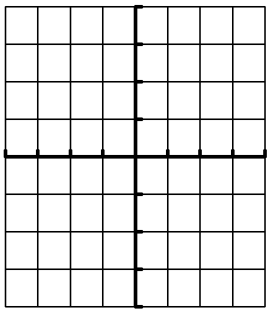
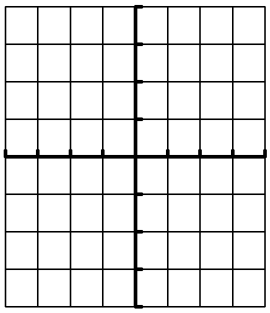
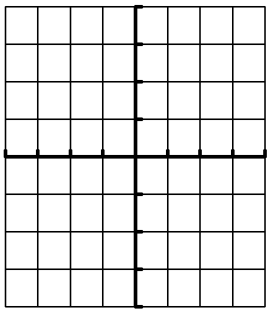
a) $10^2 = 100$ b) $e^0 = 1$

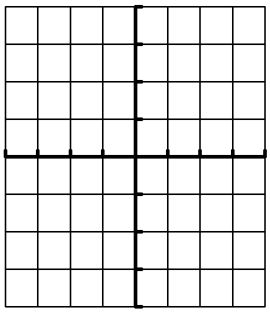
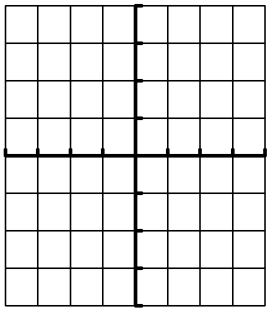
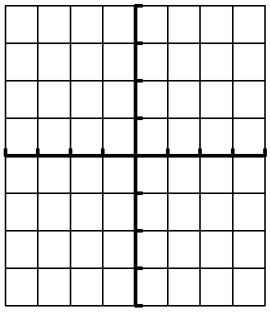
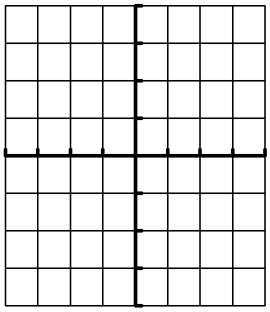
17. Use the Unit Circle to fill in the Chart. Give exact, reduced, rationalized answers.

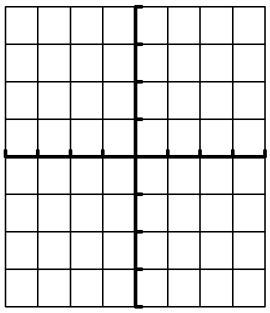
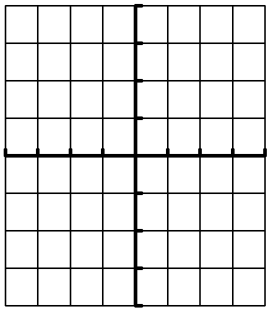
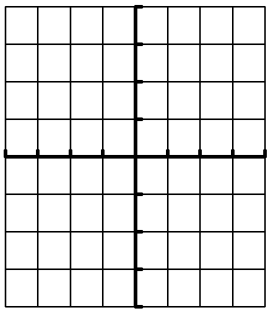
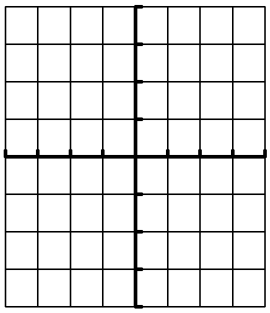
	sin (Θ)	cos (Θ)	tan (Θ)	csc (Θ)	sec (Θ)	cot (Θ)
0						
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						
$\frac{2\pi}{3}$						
$\frac{3\pi}{4}$						
$\frac{5\pi}{6}$						
π						
$\frac{7\pi}{6}$						
$\frac{5\pi}{4}$						
$\frac{4\pi}{3}$						
$\frac{3\pi}{2}$						
$\frac{5\pi}{3}$						
$\frac{7\pi}{4}$						
$\frac{11\pi}{6}$						
2π						

18. Sketch a graph of the following functions and state the domain and range of each.

Function	Graph	Domain	Range
$f(x) = x$	 tick mark scale: x-axis_____ y-axis_____		

$f(x) = x^2$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = x^3$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \lvert x \rvert$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \sqrt{x}$	 tick mark scale: x-axis_____ y-axis_____		

$f(x) = \frac{1}{x}$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \frac{1}{x^2}$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \sin x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \cos x$	 tick mark scale: x-axis_____ y-axis_____		

$f(x) = \tan x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = e^x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \ln x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = [x]$	 tick mark scale: x-axis_____ y-axis_____		

19. Solve each equation for x. Provide exact answers.

a) $\log_x 5 = -1/3$ b) $\log_{10}(2x+5) = \log_{10}(5x - 4)$ c) $\log_3(4x + 5) - \log_3(3 - 2x) = 2$

d) $4^{x^2-1} = 12$ e) $2^x = 3^{x+3}$ f) $\log_{\frac{3}{4}}\left(\frac{27}{64}\right) = x$

20. Assume that x is positive, use properties of logarithms to write the expression as a single logarithm.

a) $\frac{8}{5}\log_n 2x^2 + \frac{2}{3}\left(\log_n 2 + \log_n x^2\right)$ b) $\frac{1}{6}\log_4 x^{10} + \frac{1}{8}\log_4 x^4 - \frac{1}{12}\log_4 x$.

21. Assume that x is positive, use properties of logarithms to write the expression as a sum or difference of logarithms.

a) $\ln\left(\frac{\sqrt[5]{30}}{x^3y^2}\right)$ b) $\ln\left(\sqrt[6]{\frac{\sqrt[3]{4x^9}}{z^8}}\right)$

22. Evaluate the expression without using a calculator.

a) $\left(3^2\right)^{\log_9 8}$ b) $e^{\ln\sqrt[4]{e}}$ c) $10^{\log\sqrt{43}}$ d) $\ln e^{931}$

e) ln e f) ln 1

23. a) Find the discriminant of $x^2 + 2x - 2 = 0$.

b) Describe the nature of the roots of the equation.

c) Solve the equation by using the Quadratic Formula.

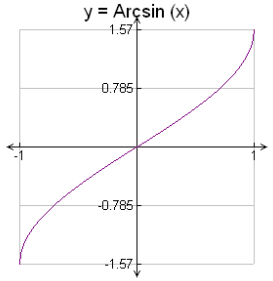
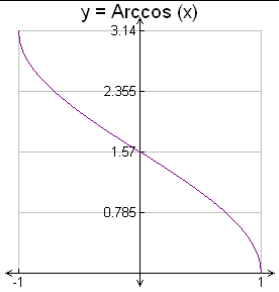
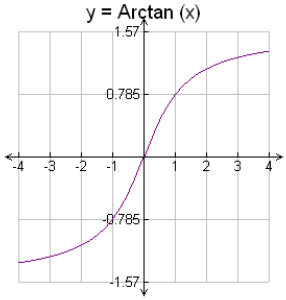
24. a) Find the discriminant of $2x^2 + 3x + 5 = 0$

b) Describe the nature of the roots of the equation.

c) Solve the equation by using the Quadratic Formula.

Please submit to Google classroom by midnight on Friday, July 21

Inverse Trigonometric Functions

Function	Graph	Domain	Range
y = arcsin (x) or $\sin^{-1}(x)$		$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
y = arccos (x) or $\cos^{-1}(x)$		$-1 \le x \le 1$	$0 \le y \le \pi$
y = arctan (x) or $\tan^{-1}(x)$		All reals	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

By restricting the domains and ranges, the inverse trig functions are actually functions.

25. Evaluate the following. Remember you are in RADIAN mode. Give exact answers and don't forget the restrictions as stated in the above table.

a) $\sin^{-1}(1) =$ _____ b) $\cos^{-1}(-1) =$ _____ c) $\tan^{-1}\left(-\sqrt{3}\right) =$ _____

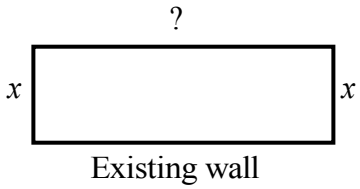
d) $\tan^{-1}(-1) =$ _____ e) $\cos^{-1}\left(\frac{1}{2}\right) =$ _____ f) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ _____

g) $\tan\left(\cos^{-1}\frac{5}{13}\right) =$ _____ h) $\tan^{-1}(\cos(0)) =$ _____ i) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right) =$ _____

j) $\cos\left(\cos^{-1}\frac{1}{2}\right) =$ _____ k) Find the algebraic expression equivalent to $\sin\left(\tan^{-1}\left(\frac{2x+3}{5x}\right)\right)$. _____

l) If the graph of f(x) has the point (2, 7) then what is one point that will be on the graph of $f^{-1}(x)$?

26. Three sides of a fence and an existing wall form a rectangular enclosure. The total length of a fence used for the three sides is 240 ft. Let x be the length of two sides perpendicular to the wall as shown. Write an equation of area A of the enclosure as a function of the length x of the rectangular area as shown in the above figure. The find value(s) of x for which the area is 5500 ft^2.



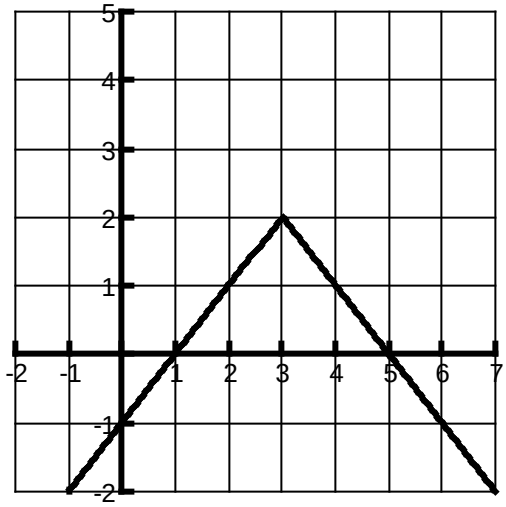
27. Arturo invests $2700 in a savings account that pay 9% interest, compounded continuously.

a. If there are no other transactions, when will his balance reach $4550?

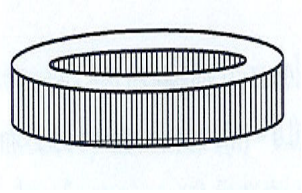
b. How long will it take the investment to double?

28. Solve the inequality $x^2 - x - 12 > 0$ Use interval notation.

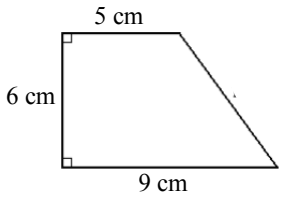
29. Find a formula for the graphed function f(x)= ? Write f(x) as both an absolute value function and piecewise with two linear functions.

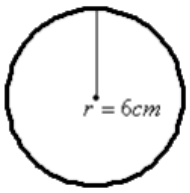


30. The figure below is the shape of a washer with an outer radius of 18 ft and an inner radius of 15 ft. If the figure is 3 ft in height, find the volume of the washer.



31. Find the area of the figures below.





32. a. A car travels 360 miles in a period of 180 minutes. Find the average velocity of the car in miles per hour over this time period.

b. In 1984, the Fizzy Cola Company sold 23 million gallons of soda. By 2003, the company was selling 127 million gallons of soda. What is the average rate of change in the number of gallons of soda per year?

c. During a recent trip to the store, a car's velocity went from 0 to 60 mph in 20 seconds. What is the average acceleration of the car in miles per hour per hour?

B

B

33. A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface are of the top of the water?

34. State the factoring rule for the following:

a) Sum of Two Squares: $a^2 + b^2 =$ _____

b) Difference of Two Squares: $a^2 - b^2 =$ _____

c) Difference of Two Cubes: $a^3 - b^3 =$ _____

d) Sum of Two Cubes: $a^3 + b^3 =$ _____

Please submit to Google classroom by midnight on Friday, August 4

LIMITS!

First of all, watch some explanations of limits online. Go to the following website http://www.calculus-help.com/tutorials and watch lessons 1-5. I've included some questions below that go with each lesson. Then answer the corresponding questions after watching each lesson.

Lesson 1: What is a limit?

1. How would you describe a limit?

2. Some graphs are straightforward, like $f(x) = x^2$. What's $\lim_{x \to 3} x^2$?

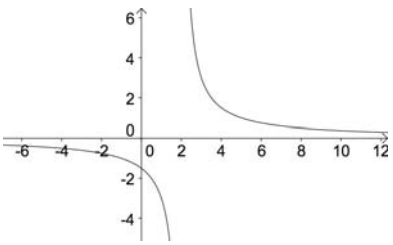
3. Some graphs are more 'mysterious', like $f(x) = \dfrac{x^2 + 3x - 4}{x - 1}$. What's $\lim_{x \to 1} \dfrac{x^2 + 3x - 4}{x - 1}$?

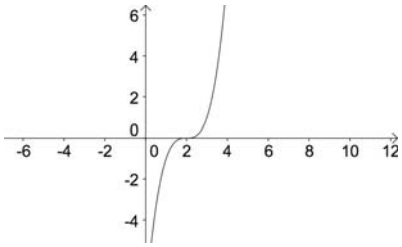
   (Either find the limit, if you know how, or describe how to find it based on the explanation you saw online)

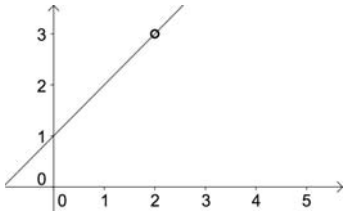
Lesson 2: When does a limit exist?

4. How is a limit like two friends meeting at a diner?

5. Look at the following graphs. Which one(s) have a limit that exists at x=2, and which one(s) don't have a limit that exists at x=2?

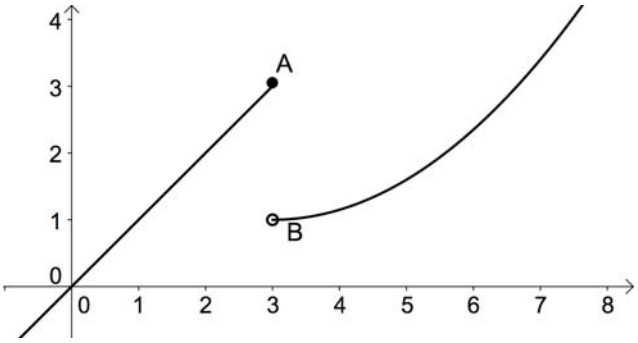






6. What is meant by a "right hand limit" and a "left hand limit"?

7. For the following graph, find: $\lim_{x\to 3^+} f(x) =$ $\lim_{x\to 3^-} f(x) =$



8. For a limit to exist, what has to be true for the left hand and the right hand limits?

Lesson 3: How do you evaluate limits?

9. What are the 3 methods for evaluating limits?

10. When can you use the substitution method?

11. When can you use the factoring method?

12. When can you use the conjugate method?

13. Figure out which method to use for the following limits, and evaluate them:

a. $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} =$

b. $\lim_{x\to 1} \frac{4x+5}{6x-1} =$

c. $\lim_{x\to 1} \frac{x^2+3x-4}{x-1} =$

Lesson 4: Limits and Infinity

14. How do you know if a function has a vertical asymptote?

15. When you take the limit of a function at its vertical asymptote, the limit will be _____ or _____.

16. To determine if a function has a horizontal asymptote, look at the…

17. If the degrees of the numerator and denominator are equal, how do you find the horizontal asymptote?

18. If the degree of the denominator is greater than that of the numerator, what's the horizontal asymptote?

19. If the degree of the denominator is less than that of the numerator, what's the horizontal asymptote?

20. If we say that the limit of a function EQUALS INFINITY, this really means that….

Lesson 5: Continuity

21. What does it mean for a function to be continuous?

22. What are the 3 types of discontinuity? Draw an example of a graph of each kind below:

23. In order to be continuous, 3 things must be true:
    - There must be no _____
    - There must be no _____
    - The limit must be equal to the _____

24. An "easy" way to tell if a function is continuous is this: